Logic and the Methodology of Science

The Group in Logic and the Methodology of Science offers an interdisciplinary program of study and research leading to the PhD degree. Students in the program acquire a good understanding of the mathematical theory known as Mathematical Logic, which deals in a rigorous way with such central concepts as truth, definability, provability, and computability. They may then seek to contribute to this theory or to apply it. There are important areas of application in mathematics, philosophy, computer science, and elsewhere.

The program is administered by an interdepartmental group that cooperates closely with the Computer Science Division, the Department of Mathematics, and the Department of Philosophy.

Admission to the University

Applying for Graduate Admission

Thank you for considering UC Berkeley for graduate study! UC Berkeley offers more than 120 graduate programs representing the breadth and depth of interdisciplinary scholarship. A complete list of graduate academic departments, degrees offered, and application deadlines can be found on the Graduate Division website (http://grad.berkeley.edu/programs/list/).

Prospective students must submit an online application to be considered for admission, in addition to any supplemental materials specific to the program for which they are applying. The online application can be found on the Graduate Division website (http://grad.berkeley.edu/admissions/).

Admission Requirements

The minimum graduate admission requirements are:

1. A bachelor's degree or recognized equivalent from an accredited institution;
2. A satisfactory scholastic average, usually a minimum grade-point average (GPA) of 3.0 (B) on a 4.0 scale; and
3. Enough undergraduate training to do graduate work in your chosen field.

For a list of requirements to complete your graduate application, please see the Graduate Division's Admissions Requirements page (https://grad.berkeley.edu/admissions/steps-to-apply/requirements/). It is also important to check with the program or department of interest, as they may have additional requirements specific to their program of study and degree. Department contact information can be found here (http://guide.berkeley.edu/graduate/degree-programs/).

Where to apply?

Visit the Berkeley Graduate Division application page (http://grad.berkeley.edu/admissions/apply/).

Admission to the Program

For admission to the graduate program, students must have completed an undergraduate major in philosophy, mathematics, science, or some related field. Their course of study should include at least one full year of upper division logic, one full year of upper division mathematics other than logic, one full year of upper division philosophy, and one upper division course in some science. Exceptions to these requirements are permitted at the discretion of the head graduate adviser.

As in most PhD programs at Berkeley, the work in this program is divided into two phases. In the first, the student acquires a fairly broad but rigorous working knowledge in three areas. His or her competence in these areas is tested in a two-part comprehensive preliminary examination and a qualifying examination. Part I of the preliminary examination deals with the foundations of mathematics (including elements of model theory, recursion theory, and incompleteness and undecidability results). Part II concerns one of the following areas of philosophy: philosophy of science, philosophy of language, philosophy of mathematics, or philosophical logic. The qualifying examination covers material from a mathematics option, a philosophy option, or a special option (for details, see Preliminary Examination (p. 1) and Qualifying Examination (p. 2) below). Students are also required to take two 4-unit courses, Math 225A and 225B (Metamathematics), usually in the first year of the program. In the second phase of the PhD program, after having passed the qualifying examination, the student selects a dissertation supervisor and, under his or her guidance, carries out original research and writes a dissertation. Since most of the faculty have strong interests in logic, students wishing to work in some other area of the methodology of science may find difficulty in finding a dissertation supervisor unless they propose to approach their problems using the methods of mathematics and logic.

Advancement to Candidacy

For advancement to candidacy for the PhD degree, the student must complete the requirements described in Independent Work (p. 1), Preliminary Examination (p. 1), and Qualifying Examination (p. 2), below, and must arrange for a faculty member of the group to serve as his or her dissertation supervisor.

Independent Work

Each student must give evidence of capacity to work independently. As regards philosophy, each student must successfully complete a one-semester long philosophy seminar. The seminar must call for the student's active participation, involving both (i) the oral exposition of assigned papers and topics and (ii) the completion of a term paper. This requirement is to be completed prior to the appointment of the committee for the qualifying examination. As regards mathematics, each student who chooses the mathematics option of the qualifying examination must take a course or seminar in mathematics that involves the oral exposition of assigned papers and topics.

Preliminary Examination

The preliminary examination consists of two separate parts, which are usually taken on different days. The two parts can be taken in any order, however, Part II may only be taken after passing the philosophy seminar requirement.

Part I. This is a three hour written examination in the foundations of mathematics covering roughly the standard topics usually included in MATH 225A /MATH 225B (Metamathematics). The foundations exam is given once a year, usually in June. An additional exam may be offered in December. Logic graduate students are required to take these two courses for credit, which is usually done in the student's first year in

the program. Exceptions to this course requirement may be granted by petition.

Sample topics include (but are not limited to) the following:

- *First-order logic:* Completeness theorem, compactness theorem, preservation theorems, Löwenheim-Skolem theorems, complete theories, decidable theories. Elementary and pseudo-elementary classes, elementary equivalence and elementary extensions, Skolem functions, characterization of universal classes, interpolation and Beth's definability theorem. Applications such as dense linear orderings and algebraically closed fields.
- *Incompleteness and undecidability:* Recursive and recursively enumerable sets, the arithmetic hierarchy. Interpretability between theories. Applications to undecidability of theories. Gödel's incompleteness theorem. Formalized arithmetic and Gödel's theorem on consistency proofs.

Students are expected to be fully conversant with basic notions in logic at the level of an undergraduate mathematical logic courses as exposed in textbooks such as Enderton's "A Mathematical Introduction to Logic." Students are not expected to have expertise in set theory, but they should understand it somewhat beyond the undergraduate level (as represented, for example, in Enderton's "Elements of Set Theory"). The first part (through chapter 13 on the constructible universe) of Jech's "Set Theory" contains the requisite material. For recursion theory, Soare's book "Recursively Enumerable Sets and Degrees" through parts A and B is a good reference. For Gödel's incompleteness theorems and the theory of Peano arithmetic, Kaye's book "Models of Peano Arithmetic" is an excellent source. For model theory, there are several good books, all of which bear slight variants of the title "Model Theory" by Chang and Keisler, Hodges, Marker, and Poizat.

It is important to recognize that these texts include material which goes beyond what would be expected on the preliminary examination. On the other hand, the questions for the examination will not be derived exclusively from these sources. Students should consult with the committee in charge of the preliminary examination during the preceding semester for advice about a more detailed preparatory course of study and readings.

Past examinations are archived online (http://logic.berkeley.edu/past-prelims.html).

Part II. This is a three hour written examination in one of the following fields:

- Philosophy of mathematics and logic
- Philosophical logic (broadly construed to include formal epistemology, probability, decision theory, and game theory)
- Philosophy of science
- Philosophy of language

A student may take this part of the preliminary examination only after completing the philosophy seminar requirement as described in Independent Work (p. 1). Students may wish to take an independent study course with a Logic Group philosophy professor to prepare for this part of the examination, and this can be done concurrently with the philosophy seminar. Other resources for preparing for the examination include upper division undergraduate courses and graduate seminars which the Philosophy Department regularly offers in all of these fields.

The student's plan of study must be within one of the four areas listed, and must be agreed to by the examination committee well before the date is set for the examination. An archive of preapproved syllabi, some of which coordinate with course offerings in the Philosophy Department, is available here (http://logic.berkeley.edu/philosophy-prelims.html). A student may wish instead to submit a prospectus for a special course of study, which would include a list of readings and a brief description of the topics to be covered in the examination. This option requires approval of the committee.

Procedural Matters

For Part I: A three-member examining committee will be appointed by the Chair of the Group in Logic and the Methodology of Science to administer the first part of the preliminary examination. The chair of the committee must be a member of the group. The date of the examination will be set by the chair of the group. This examination will usually be offered only once each academic year, in June. If in a given year it is deemed necessary, an additional examination will be scheduled for January.

For Part II: A (second) three-member examining committee will be selected by the student subject to approval by the Chair of the Group in Logic and the Methodology of Science. This committee must consist of at least two members of the Philosophy Department, and the chair of the committee must be a member of the group. The examination date may be chosen by the student, subject to the approval of the chair of the committee. The committee will prepare a list of six questions that shall constitute the written exam. The examination will be graded by the committee members on a pass/non-pass basis.

If in any of the two parts of the preliminary examination the student is deemed to have failed that part on the first try, the student may be request re-examination. A third attempt, for either part, is not permitted. A student failing any part of the preliminary examination must consult promptly with the graduate adviser.

Qualifying Examination

The oral qualifying examination is held on one day and is normally two to three hours in length. For the qualifying examination the student may choose one of the following three options; no matter which option is chosen, the Graduate Council requires that three subject areas be entered on the application for the qualifying examination that is due in the Graduate Division for processing at least three weeks before the exam.

Mathematics Option

The oral qualifying examination is held on one day and is normally two to three hours in length. At last three subject areas must be covered during the examination. The student may select either all of the material in analysis and about half of the material in algebra (either general algebra, groups, and rings or general algebra, vector spaces and modules, and fields) or all of the material in algebra and about half of the material in analysis (either general topology or measure and integration).

Analysis

- *General topology:* Metric spaces, completeness. Topological spaces, bases, continuous functions, subspaces, product spaces, quotient spaces. Connectedness, separability, Hausdorff spaces. Compactness and Tychonov's theorem. Subspaces and continuous functions for compact spaces. Zero-dimensional spaces and Stone's representation theorem for Boolean algebras. Meager sets, property of Baire, Baire category theorem, and the Kuratowski-Ulam theorem. Convergence of nets. This material can be found in Royden (Section

2.7, Chapters 7-8, and Sections 9.1-9.6) and Oxtoby (Chapters 1, 4, 8, 9, 12, 15). Much of the material can also be found in the corresponding parts of Chapters 1, 2, 3, 5 of Kelley, in expanded form in Munkres, and in advanced form in Dugundji. Most (but not quite all) of the material is normally covered in MATH 202A (Topology and Analysis).

- *Measure and integration:* Lebesgue measure and integration in Rn. Construction of measure and integral. Monotone convergence, dominated convergence, absolute continuity, Radon-Nikodym theorem, Fubini's theorem, Egorov's theorem. Elements of general measure and integration theory. Borel spaces. Product measures. This material can be found in Rudin (Chapter 11 through p. 325), Bartle (Chapters 1-5, 7-8, 10), and Oxtoby (Chapters 1, 3, 8). Much of the material can also be found in Royden (Chapters 3-4 and Sections 11.1-11.6), Halmos (parts of Chapters 1-7), and Burkill. Most (but not quite all) of the material is normally covered in MATH 105 (Second Course in Analysis).

Algebra

- *General algebra:* Subalgebras, homomorphisms, congruences, and quotient algebras. Direct and subdirect products, free algebras and varieties. The material can be found in Grätzer (Chapters 7, 11, 19, and Jacobson (Section 1.11). Some of the material can also be found in Malcev (Section 2).
- *Groups:* Permutation and alternating groups. Normal subgroups, abelian groups and finitely generated abelian groups. Sylow theorems. The material can be found in Herstein (Sections 1.3, 2.1-2.11, 2.13-2.14). The material can also be found in Jacobson (Sections 1.1-1.12), Rotman (Chapters 1–4 through p. 60), and Sah (Chapters III 1, 2, 4, 5).
- *Rings:* Ideals and quotient rings, integral domains, Euclidean rings, polynomial rings, unique factorization domains. The material can be found in Herstein (Chapter 3). The material can also be found in Sah (Chapter IV), Zariski-Samuel (parts of Chapter I), and Lang (Chapters I and V).
- *Vector spaces and modules:* Basic concepts. Linear independence and bases. Modules. The material can be found in Herstein (Sections 4.1, 4.2, 4.5). The material can also be found in Sah, Zariski-Samuel, and Lang.
- *Fields:* Algebraic extensions, splitting field, separable extensions, basic ideas of Galois theory, transcendental extensions and degree of transcendence, real closed and algebraically closed fields. The material can be found in Herstein (Sections 5.1, 5.3-5.8) and van der Waerden (Sections 62-64, 66-71).

Philosophy Option

An oral examination in a field chosen from one of the four areas listed under Part II above, or (with permission from the graduate adviser) from some other area of philosophy. The graduate adviser must judge the scope and the intended reading list to be sufficiently different from that of the student's Part II examination. The exam must conform to the university rules and regulations for qualifying examinations. In particular, the examination must cover at least three topics.

Special Option

A special examination based on the needs of the individual student. In this case the student must submit a proposed syllabus and then obtain written approval of the graduate adviser and a faculty member of the group (who thereby expresses willingness to become the student's dissertation supervisor); such approval is only to be given after circulation of the student's proposal among all group faculty members for comment. The exam must conform to the university rules and regulations for qualifying examinations. In particular, the examination must be an oral examination covering at least three topics.

Procedural Matters

For the qualifying examination, a four-member examining committee will be appointed by the dean of the Graduate Division upon recommendation of the graduate adviser. Committee members must be members of Berkeley's Academic Senate and the Chair must be a member of the Logic Group.

For the philosophy option of qualifying examination, the student must submit to the examination committee a prospectus outlining the scope of the oral examination. The prospectus should include a reading list and a brief description of the topics to be covered in the examination. Prior to approval of the prospectus, the chair of this committee may consult with the faculty member of the Department of Philosophy who is in charge of the department's qualifying examination concerning the reading list and topics to be covered. (If the prospectus submitted is judged to be acceptable by the committee, a copy will be put in a file in the group office for purposes of guiding other students and examination committees.) The prospectus should be submitted to the committee well in advance of the examination date. If the student is judged to have failed the qualifying examination on the first try, the committee may recommend that the student be granted a reexamination following a reasonable delay, usually of three months. A third attempt is not permitted. A student failing the qualifying examination must consult promptly with the graduate adviser.

Dissertation and Final Examination

After advancement to candidacy, the student must write an acceptable dissertation and pass a final oral examination in order to earn the degree. In administering this requirement the group follows Plan A as outlined in the Berkeley Academic Guide in the section entitled "The Doctoral Dissertation."

Time Limits

The Graduate Division together with the group has imposed the following requirements to help regulate the progress of students in the program toward the PhD.

Each student must 1) pass both parts of the preliminary examination before the beginning of the fifth semester, 2) attempt one of them before the beginning of the third semester, and 3) attempt both of them before the beginning of the fourth semester. Permission for an extension of these time limits will only be granted in special circumstances and requires the written permission of the chair and graduate advisor.

Each student must pass the qualifying examination within three calendar years after entering the program, again unless explicit written permission is granted by the chair and graduate adviser for an extension of this time limit.

These are maximum time limits. All students are encouraged to take both parts of the preliminary examination and the qualifying examination as soon as possible and in fact, many students will take them much sooner than required by the time limits given above. (The graduate advisor can in individual cases, e.g., for students entering with relevant prior graduate

study at Berkeley or elsewhere, insist that the examinations be taken within stricter time limits.) Students must be advanced to candidacy for the PhD no later than the semester following the one in which the qualifying examination was passed.

Students must complete all requirements for the PhD degree within four years after advancement to candidacy. Students who do not obtain the PhD within this four-year period will no longer be considered in candidacy for the PhD unless special action is taken by the group.