

# Logic and the Methodology of Science

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The Group in Logic and the Methodology of Science offers an interdisciplinary program of study and research leading to the PhD degree. Students in the program acquire a good understanding of the mathematical theory known as Mathematical Logic, which deals in a rigorous way with such central concepts as truth, definability, provability, and computability. They may then seek to contribute to this theory or to apply it. There are important areas of application in mathematics, philosophy, computer science, and elsewhere.

The program is administered by an interdepartmental group which cooperates closely with the Computer Science Division, the Department of Mathematics, and the Department of Philosophy.

## Admission to the University

### Minimum Requirements for Admission

The following minimum requirements apply to all graduate programs and will be verified by the Graduate Division:

1. A bachelor's degree or recognized equivalent from an accredited institution;
2. A grade point average of B or better (3.0);
3. If the applicant comes from a country or political entity (e.g., Quebec) where English is not the official language, adequate proficiency in English to do graduate work, as evidenced by a TOEFL score of at least 90 on the iBT test, 570 on the paper-and-pencil test, or an IELTS Band score of at least 7 on a 9-point scale (note that individual programs may set higher levels for any of these); and
4. Sufficient undergraduate training to do graduate work in the given field.

### Applicants Who Already Hold a Graduate Degree

The Graduate Council views academic degrees not as vocational training certificates, but as evidence of broad training in research methods, independent study, and articulation of learning. Therefore, applicants who already have academic graduate degrees should be able to pursue new subject matter at an advanced level without the need to enroll in a related or similar graduate program.

Programs may consider students for an additional academic master's or professional master's degree only if the additional degree is in a distinctly different field.

Applicants admitted to a doctoral program that requires a master's degree to be earned at Berkeley as a prerequisite (even though the applicant already has a master's degree from another institution in the same or a closely allied field of study) will be permitted to undertake the second master's degree, despite the overlap in field.

The Graduate Division will admit students for a second doctoral degree only if they meet the following guidelines:

1. Applicants with doctoral degrees may be admitted for an additional doctoral degree only if that degree program is in a general area of knowledge distinctly different from the field in which they earned their original degree. For example, a physics PhD could be admitted to a doctoral degree program in music or history; however, a student with

a doctoral degree in mathematics would not be permitted to add a PhD in statistics.

2. Applicants who hold the PhD degree may be admitted to a professional doctorate or professional master's degree program if there is no duplication of training involved.

Applicants may apply only to one single degree program or one concurrent degree program per admission cycle.

## Required Documents for Applications

1. **Transcripts:** Applicants may upload *unofficial* transcripts with your application for the departmental initial review. *If the applicant is admitted*, then *official* transcripts of all college-level work will be required. Official transcripts must be in sealed envelopes as issued by the school(s) attended. If you have attended Berkeley, upload your unofficial transcript with your application for the departmental initial review. *If you are admitted*, an official transcript with evidence of degree conferral *will not* be required.
2. **Letters of recommendation:** Applicants may request online letters of recommendation through the online application system. Hard copies of recommendation letters must be sent directly to the program, not the Graduate Division.
3. **Evidence of English language proficiency:** All applicants from countries or political entities in which the official language is not English are required to submit official evidence of English language proficiency. This applies to applicants from Bangladesh, Burma, Nepal, India, Pakistan, Latin America, the Middle East, the People's Republic of China, Taiwan, Japan, Korea, Southeast Asia, most European countries, and Quebec (Canada). However, applicants who, at the time of application, have already completed at least one year of full-time academic course work with grades of B or better at a US university may submit an official transcript from the US university to fulfill this requirement. The following courses will not fulfill this requirement:
  - courses in English as a Second Language,
  - courses conducted in a language other than English,
  - courses that will be completed after the application is submitted, and
  - courses of a non-academic nature.

If applicants have previously been denied admission to Berkeley on the basis of their English language proficiency, they must submit new test scores that meet the current minimum from one of the standardized tests. Official TOEFL score reports must be sent directly from Educational Test Services (ETS). The institution code for Berkeley is 4833. Official IELTS score reports must be mailed directly to our office from the British Council. TOEFL and IELTS score reports are only valid for two years.

## Where to Apply

Visit the Berkeley Graduate Division application page (<http://grad.berkeley.edu/admissions/apply>).

## Admission to the Program

For admission to the graduate program, students must have completed an undergraduate major in philosophy, mathematics, science, or some related field. Their course of study should include at least one full year of upper division logic, one full year of upper division mathematics other than logic, one full year of upper division philosophy, and one upper

division course in some science. Exceptions to these requirements are permitted at the discretion of the head graduate adviser.

As in most PhD programs at Berkeley, the work in this program is divided into two phases. In the first, the student acquires a fairly broad but rigorous working knowledge in three areas. His or her competence in these areas is tested in a two-part comprehensive preliminary examination and a qualifying examination. Part I of the preliminary examination deals with the foundations of mathematics (including elements of model theory, recursion theory, and incompleteness and undecidability results). Part II concerns one of the following areas of philosophy: philosophy of science, philosophy of language, philosophy of mathematics, or philosophical logic. The qualifying examination covers material from a mathematics option, a philosophy option, or a special option (for details, see Preliminary Examination (p. ) and Qualifying Examination (p. ) below). In the second phase of the PhD program, after having passed the qualifying examination, the student selects a dissertation supervisor and, under his or her guidance, carries out original research and writes a dissertation. Since most of the faculty have strong interests in logic, students wishing to work in some other area of the methodology of science may find difficulty in finding a dissertation supervisor unless they propose to approach their problems using the methods of mathematics and logic.

## Advancement to Candidacy

For advancement to candidacy for the PhD degree, the student must complete the requirements described in Independent Work (p. ), Preliminary Examination (p. ), and Qualifying Examination (p. ), below, and must arrange for a faculty member of the group to serve as his or her dissertation supervisor.

## Independent Work

Each student must give evidence of capacity to work independently. As regards philosophy, each student must successfully complete a one semester long philosophy seminar. The seminar must call for the student's active participation, involving both (i) the oral exposition of assigned papers and topics and (ii) the completion of a term paper. This requirement is to be completed prior to the appointment of the committee for the qualifying examination. As regards mathematics, each student who chooses the mathematics option of the qualifying examination must take a course or seminar in mathematics that involves the oral exposition of assigned papers and topics.

## Preliminary Examination

The preliminary examination consists of two separate parts, which are usually taken on different days. The two parts can be taken in any order, however Part II may only be taken after passing the philosophy seminar requirement.

**Part I.** This is a three hour written examination in the foundations of mathematics covering roughly the standard topics usually included in MATH 225A /MATH 225B (Metamathematics). The foundations exam is given once a year, usually in June. An additional exam may be offered in December. Logic graduate students are required to take these two courses for credit, which is usually done in the student's first year in the program. Exceptions to this course requirement may be granted by petition.

Sample topics include (but are not limited to) the following:

- *First-order logic:* Completeness theorem, compactness theorem, preservation theorems, Löwenheim-Skolem theorems, complete

theories, decidable theories. Elementary and pseudo-elementary classes, elementary equivalence and elementary extensions, Skolem functions, characterization of universal classes, interpolation and Beth's definability theorem. Applications such as dense linear orderings and algebraically closed fields.

- *Incompleteness and undecidability:* Recursive and recursively enumerable sets, the arithmetic hierarchy. Interpretability between theories. Applications to undecidability of theories. Gödel's incompleteness theorem. Formalized arithmetic and Gödel's theorem on consistency proofs.

Students are expected to be fully conversant with basic notions in logic at the level of an undergraduate mathematical logic courses as exposed in textbooks such as Enderton's "A Mathematical Introduction to Logic." Students are not expected to have expertise in set theory, but they should understand it somewhat beyond the undergraduate level (as represented, for example, in Enderton's "Elements of Set Theory"). The first part (through chapter 13 on the constructible universe) of Jech's "Set Theory" contains the requisite material. For recursion theory, Soare's book "Recursively Enumerable Sets and Degrees" through parts A and B is a good reference. For Gödel's incompleteness theorems and the theory of Peano arithmetic, Kaye's book "Models of Peano Arithmetic" is an excellent source. For model theory, there are several good books, all of which bear slight variants of the title "Model Theory" by Chang and Keisler, Hodges, Marker, and Poizat.

It is important to recognize that these texts include material which goes beyond what would be expected on the preliminary examination. On the other hand, the questions for the examination will not be derived exclusively from these sources. Students should consult with the committee in charge of the preliminary examination during the preceding semester for advice about a more detailed preparatory course of study and readings.

Past examinations are archived online (<http://logic.berkeley.edu/past-prelims.html>).

**Part II.** This is a three hour written examination in one of the following fields:

- Philosophy of mathematics and logic
- Philosophical logic (broadly construed to include formal epistemology, probability, decision theory, and game theory)
- Philosophy of science
- Philosophy of language

A student may take this part of the preliminary examination only after completing the philosophy seminar requirement as described in Independent Work (p. ). Students may wish to take an independent study course with a Logic Group philosophy professor to prepare for this part of the examination, and this can be done concurrently with the philosophy seminar. Other resources for preparing for the examination include upper division undergraduate courses and graduate seminars which the Philosophy Department regularly offers in all of these fields.

The student's plan of study must be within one of the four areas listed, and must be agreed to by the examination committee well before the date is set for the examination. An archive of preapproved syllabi, some of which coordinate with course offerings in the Philosophy Department, is available here (<http://logic.berkeley.edu/philosophy-prelims.html>). A student may wish instead to submit a prospectus for a special course of study, which would include a list of readings and a brief description of the

topics to be covered in the examination. This option requires approval of the committee.

## Procedural Matters

**For Part I:** A three-member examining committee will be appointed by the Chair of the Group in Logic and the Methodology of Science to administer the first part of the preliminary examination. The chair of the committee must be a member of the group. The date of the examination will be set by the chair of the group. This examination will usually be offered only once each academic year, in June. If in a given year it is deemed necessary, an additional examination will be scheduled for January.

**For Part II:** A (second) three-member examining committee will be selected by the student subject to approval by the Chair of the Group in Logic and the Methodology of Science. This committee must consist of at least two members of the Philosophy Department, and the chair of the committee must be a member of the group. The examination date may be chosen by the student, subject to the approval of the chair of the committee. The committee will prepare a list of six questions which shall constitute the written exam. The examination will be graded by the committee members on a pass/non-pass basis.

If in any of the two parts of the preliminary examination the student is deemed to have failed that part on the first try, the student may be request re-examination. A third attempt, for either part, is not permitted. A student failing any part of the preliminary examination must consult promptly with the graduate adviser.

## Qualifying Examination

The oral qualifying examination is held on one day and is normally two to three hours in length. For the qualifying examination the student may choose one of the following three options; no matter which option is chosen, the Graduate Council requires that three subject areas be entered on the application for the qualifying examination that is due in the Graduate Division for processing at least three weeks before the exam.

## Mathematics Option

The oral qualifying examination is held on one day and is normally two to three hours in length. At last three subject areas must be covered during the examination. The student may select either all of the material in analysis and about half of the material in algebra (either general algebra, groups, and rings or general algebra, vector spaces and modules, and fields) or all of the material in algebra and about half of the material in analysis (either general topology or measure and integration).

## Analysis

- *General topology:* Metric spaces, completeness. Topological spaces, bases, continuous functions, subspaces, product spaces, quotient spaces. Connectedness, separability, Hausdorff spaces. Compactness and Tychonov's theorem. Subspaces and continuous functions for compact spaces. Zero-dimensional spaces and Stone's representation theorem for Boolean algebras. Meager sets, property of Baire, Baire category theorem, and the Kuratowski-Ulam theorem. Convergence of nets. This material can be found in Royden (Section 2.7, Chapters 7-8, and Sections 9.1-9.6) and Oxtoby (Chapters 1, 4, 8, 9, 12, 15). Much of the material can also be found in the corresponding parts of Chapters 1, 2, 3, 5 of Kelley, in expanded form in Munkres, and in advanced form in Dugundji. Most (but not quite all) of the material is normally covered in MATH 202A (Topology and Analysis).

- *Measure and integration:* Lebesgue measure and integration in  $\mathbb{R}^n$ . Construction of measure and integral. Monotone convergence, dominated convergence, absolute continuity, Radon-Nikodym theorem, Fubini's theorem, Egorov's theorem. Elements of general measure and integration theory. Borel spaces. Product measures. This material can be found in Rudin (Chapter 11 through p. 325), Bartle (Chapters 1-5, 7-8, 10), and Oxtoby (Chapters 1, 3, 8). Much of the material can also be found in Royden (Chapters 3-4 and Sections 11.1-11.6), Halmos (parts of Chapters 1-7), and Burkill. Most (but not quite all) of the material is normally covered in MATH 105 (Second Course in Analysis).

## Algebra

- *General algebra:* Subalgebras, homomorphisms, congruences, and quotient algebras. Direct and subdirect products, free algebras and varieties. The material can be found in Grätzer (Chapters 7, 11, 19, and Jacobson (Section 1.11). Some of the material can also be found in Malcev (Section 2).
- *Groups:* Permutation and alternating groups. Normal subgroups, abelian groups and finitely generated abelian groups. Sylow theorems. The material can be found in Herstein (Sections 1.3, 2.1-2.11, 2.13-2.14). The material can also be found in Jacobson (Sections 1.1-1.12), Rotman (Chapters 1-4 through p. 60), and Sah (Chapters III 1, 2, 4, 5).
- *Rings:* Ideals and quotient rings, integral domains, Euclidean rings, polynomial rings, unique factorization domains. The material can be found in Herstein (Chapter 3). The material can also be found in Sah (Chapter IV), Zariski-Samuel (parts of Chapter I), and Lang (Chapters I and V).
- *Vector spaces and modules:* Basic concepts. Linear independence and bases. Modules. The material can be found in Herstein (Sections 4.1, 4.2, 4.5). The material can also be found in Sah, Zariski-Samuel, and Lang.
- *Fields:* Algebraic extensions, splitting field, separable extensions, basic ideas of Galois theory, transcendental extensions and degree of transcendence, real closed and algebraically closed fields. The material can be found in Herstein (Sections 5.1, 5.3-5.8) and van der Waerden (Sections 62-64, 66-71).

## Philosophy Option

An oral examination in a field chosen from one of the four areas listed under Part II above, or (with permission from the graduate adviser) from some other area of philosophy. The graduate adviser must judge the scope and the intended reading list to be sufficiently different from that of the student's Part II examination. The exam must conform to the university rules and regulations for qualifying examinations. In particular the examination must cover at least three topics.

## Special Option

A special examination based on the needs of the individual student. In this case the student must submit a proposed syllabus and then obtain written approval of the graduate adviser and a faculty member of the group (who thereby expresses willingness to become the student's dissertation supervisor); such approval is only to be given after circulation of the student's proposal among all group faculty members for comment. The exam must conform to the university rules and regulations for

qualifying examinations. In particular the examination must be an oral examination covering at least three topics.

## **Procedural Matters**

For the qualifying examination a four-member examining committee will be appointed by the dean of the Graduate Division upon recommendation of the graduate adviser. One of the four members serves as an outside member and is not a member of the group. At least two of the other members must be members of the group. Both the chair and the outside member of the committee must be members of the Berkeley Division of the Academic Senate.

For the philosophy option of qualifying examination the student must submit to the examination committee a prospectus outlining the scope of the oral examination. The prospectus should include a reading list and a brief description of the topics to be covered in the examination. Prior to approval of the prospectus, the chair of this committee may consult with the faculty member of the Department of Philosophy who is in charge of the department's qualifying examination concerning the reading list and topics to be covered. (If the prospectus submitted is judged to be acceptable by the committee, a copy will be put in a file in the group office for purposes of guiding other students and examination committees.) The prospectus should be submitted to the committee well in advance of the examination date. If the student is judged to have failed the qualifying examination on the first try, the committee may recommend that the student be granted a reexamination following a reasonable delay, usually of three months. A third attempt is not permitted. A student failing the qualifying examination must consult promptly with the graduate adviser.

## **Dissertation and Final Examination**

After advancement to candidacy the student must write an acceptable dissertation and pass a final oral examination in order to earn the degree. In administering this requirement the group follows Plan A as outlined in the Berkeley Academic Guide in the section entitled "The Doctoral Dissertation."

## **Time Limits**

The Graduate Division together with the group have imposed the following requirements to help regulate the progress of students in the program toward the PhD.

Each student must 1) pass both parts of the preliminary examination before the beginning of the fifth semester, 2) attempt one of them before the beginning of the third semester, and 3) attempt both of them before the beginning of the fourth semester. Permission for an extension of these time limits will only be granted in special circumstances, and requires the written permission of the chair and graduate adviser.

Each student must pass the qualifying examination within three calendar years after entering the program, again unless explicit written permission is granted by the chair and graduate adviser for an extension of this time limit.

These are maximum time limits. All students are encouraged to take both parts of the preliminary examination and the qualifying examination as soon as possible and in fact many students will take them much sooner than required by the time limits given above. (The graduate adviser can in individual cases, e.g., for students entering with relevant prior graduate study at Berkeley or elsewhere, insist that the examinations be taken within stricter time limits.) Students must be advanced to candidacy

for the PhD no later than the semester following the one in which the qualifying examination was passed.

Students must complete all requirements for the PhD degree within four years after advancement to candidacy. Students who do not obtain the PhD within this four-year period will no longer be considered in candidacy for the PhD unless special action is taken by the group.